Warm-Up

CST/CAHSEE: Algebra II 24.0

Given that $f(x) = 3x^2 - 4$, and $g(x) = 2x - 6$ what is $g(f(2))$?

A -2

B 6

C 8

D 10

Review: Calculus 4.0

Given:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Find $f'(x)$ when $f(x) = 4x^2 - 7$

Current: Calculus 4.0

Power Rule for Derivatives

Given: $f(x) = x^n$

$f'(x) = nx^{n-1}$

Use the power rule to find the derivative.

$$f(x) = 4x^2 - 7$$

Other: Calculus 4.0

Use the power rule to find the derivative of the following function.

$$f(x) = 7x^3 + 8x^2 - 4x - 15$$

Using Derivatives to Graph polynomials

Objective: Graph polynomials using algebraic and calculus standards. Connect Algebra standards to Calculus.
Standards: Algebra 5.0, 7.0, 14.0 Calculus 4.0

Definition: A Critical Number of a function is a number c in the domain of the function such that either $f'(c)=0$ or $f'(c)$ Does Not Exist (DNE)

Ex:

$$f(x)=x^2+5x+6 \qquad \text{Domain } f=(-\infty,+\infty)$$

$$f'(x)=2x+5$$

$$f'=0 \text{ when } x=-\frac{5}{2} \qquad f' \text{ Exists Everywhere}$$

You Try:

$$f(x)=x^3+x^2-5x-8 \qquad \text{Domain } f=(-\infty,+\infty)$$

$$f'(x)=3x^2+2x-5$$

$$f'=0 \text{ when } (3x+5)(x-1)=0$$

$$x=-\frac{5}{3}, x=1 \qquad f' \text{ Exists Everywhere}$$

Consider the following function:

$$f(x)=x^3-4x \qquad \text{Sketch the graph.}$$

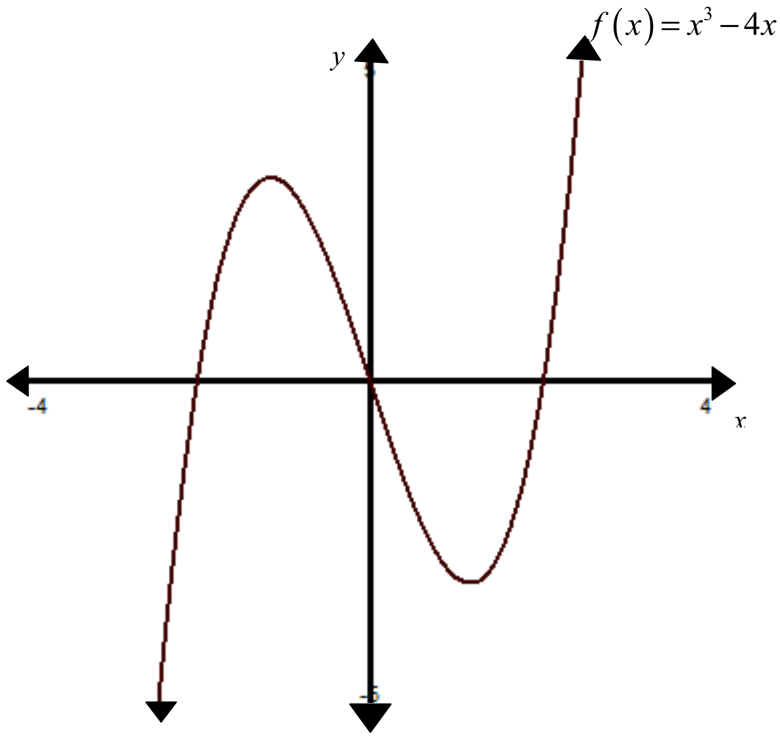
$$f(x)=x(x+2)(x-2) \qquad \text{Domain } f=(-\infty,+\infty)$$

$$f(x)=0 \text{ when } x(x+2)(x-2)=0$$

$$x=0 \text{ or } x+2=0 \text{ or } x-2=0$$

$$x=0 \text{ or } x=-2 \text{ or } x=2$$

$$x-\text{intercepts on the graph of } f$$



Ask a volunteer to sketch the graph on the board.
"So far, we have used algebraic techniques."
"Can we tell where this function is increasing or decreasing?"
"Concave up or concave down?"
"Maximum and minimum values?"
"Any thoughts?"
"Jot down ideas?"

Lets find $f'(x)$. If $f(x)=x^3-4x$

$$f'(x)=3x^2-4$$

$$f'(x)=0 \text{ when } 3x^2-4=0$$

$$3x^2-4+4=0+4$$

$$3x^2=4$$

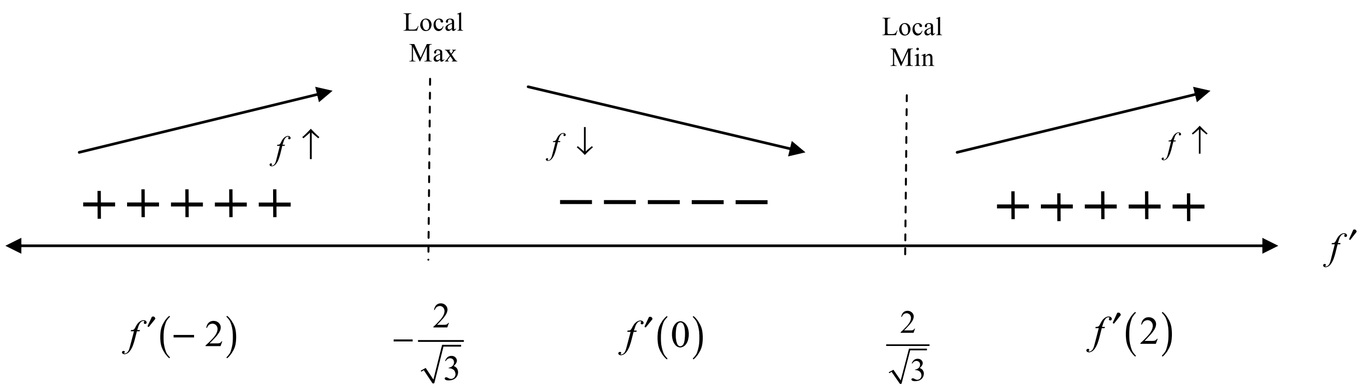
$$x^2=\frac{4}{3}$$

$$x=\pm\frac{2}{\sqrt{3}}\approx\pm1.155$$

f' Exists Everywhere

Critical Numbers

* Local Maximum & Minimums are also referred to as Relative Maximum/Minimum



<u>Key point</u>: If f has a local minimum or maximum at $x = c$ then c is a critical number of f.

If f' exists at c, then $f'(c) = 0$ (Fermat's Theorem)

<u>Definition:</u> Let f be continuous on $[a, b]$ and differentiable on (a, b)

1) If $f' > 0$ on (a, b) then f is increasing on (a, b)

2) If $f' < 0$ on (a, b) then f is decreasing on (a, b)

<u>Concavity Test:</u> The graph of a twice-differentiable function $y = f(x)$ is

1) Concave up on (a,b) if $f'' > 0$ on (a,b)

2) Concave down on (a,b) if $f'' < 0$ on (a,b)

A <u>point of inflection</u> (POI) is a point where the graph changes concavity.

Demo:

$$f(x) = x^3 - 3x^2 - 24x - 1$$

Domain $f = (-\infty, +\infty)$

$$f'(x) = 3x^2 - 6x - 24$$

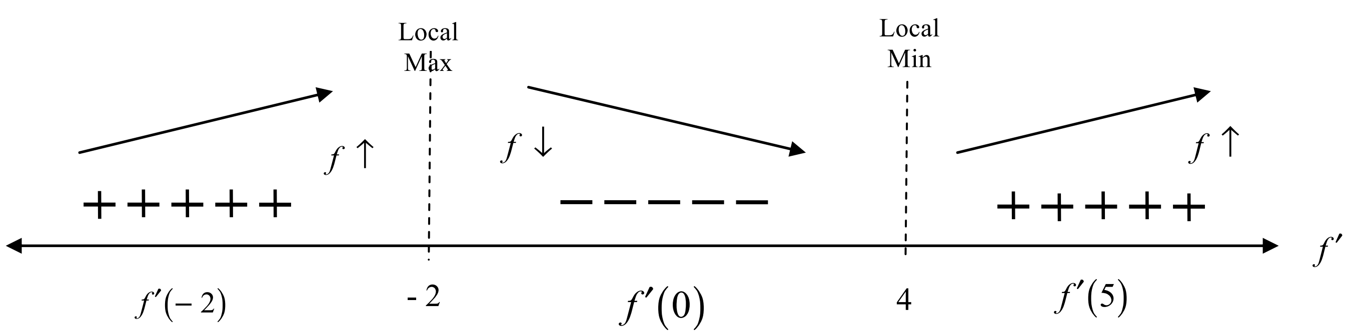
$$f'(x) = 0 \text{ when } 3x^2 - 6x - 24 = 0$$

$$3(x^2 - 2x - 8) = 0$$

$$3(x - 4)(x + 2) = 0$$

$$x = 4 \text{ or } x = -2$$

Critical Numbers f' Exists Everywhere



$f' > 0$ on $(-\infty, -2)\, \text{U}\, (4, +\infty) \therefore f$ is increasing on those intervals.

$f' < 0$ on $(-2, 4) \therefore f$ is decreasing on $(-2, 4)$.

f changes from increasing to decreasing at $x=-2 \therefore$ local maximum occurs at $x=-2$.

The local maximum value is $f(-2)=(-2)^3-3(-2)^2-24(-2)-1$
$=27$

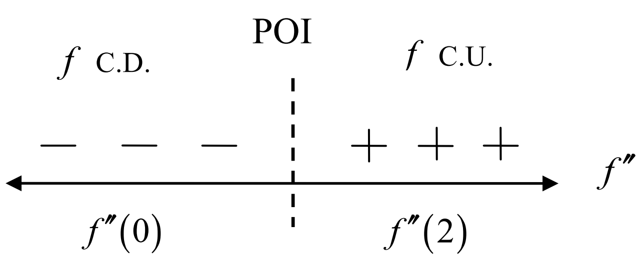
Local max at $(-2,27)$

f changes from decreasing to increasing at $x=4 \therefore$ local minimum occurs at $x=4$.

The local minimum value is $f(4)=(4)^3-3(4)^2-24(4)-1$
$=-81$

Local min at $(4,-81)$

$$f''(x)=6x-6$$
$$f''(x)=0 \text{ when } 6x-6=0$$
$$6x-6+6=0+6$$
$$6x=6$$
$$x=1$$



$f''<0$ on $(-\infty,1) \therefore f$ is concave down on $(-\infty,1)$

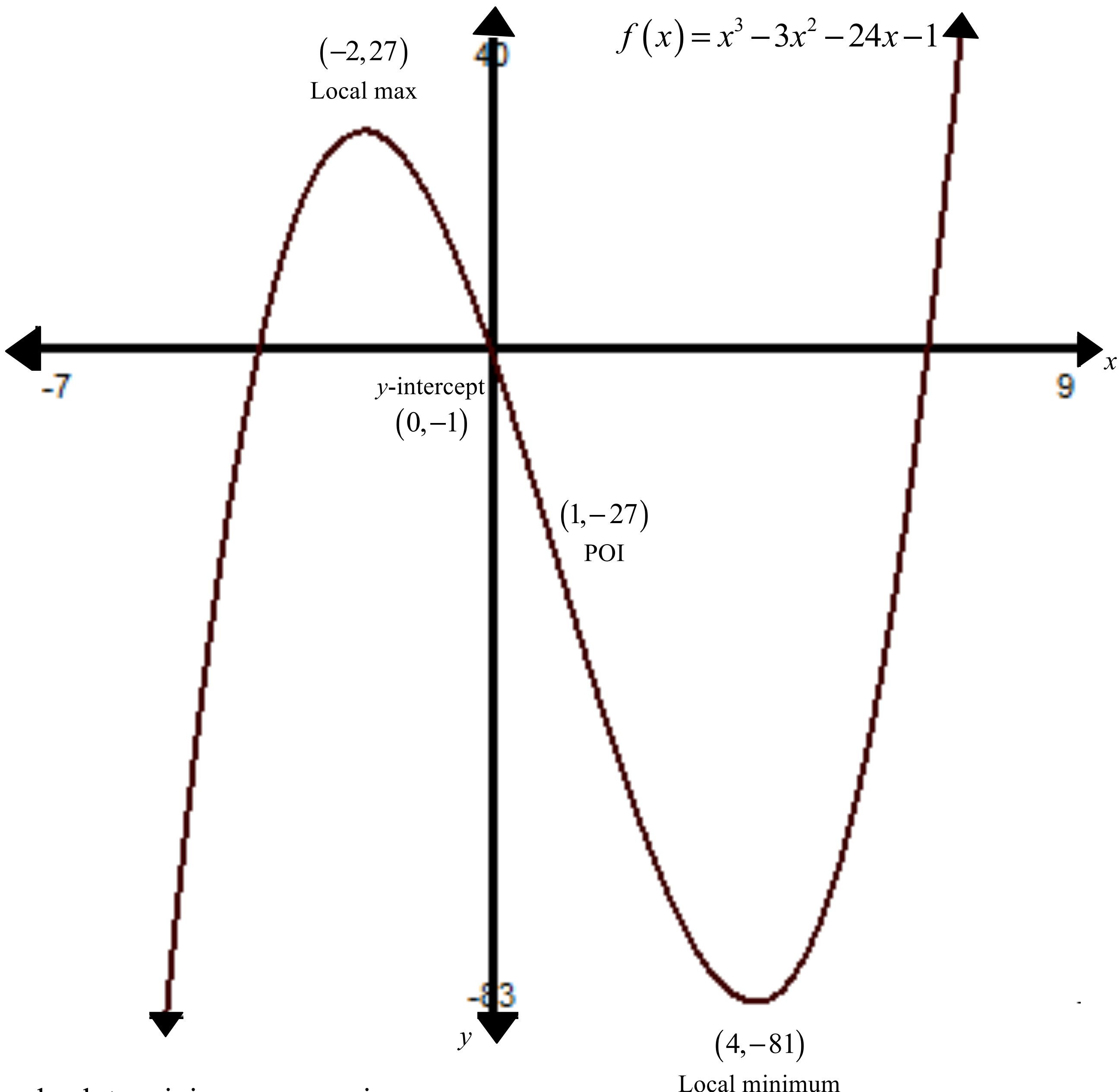
$f''>0$ on $(1,+\infty) \therefore f$ is concave up on $(1,+\infty)$

f changes concavity at $x=1 \therefore$ point of inflection occurs at $x=1$

$f(1)=(1)^3-3(1)^2-24(1)-1$
$=-27$

POI at $(1,-27)$

Do we have enough info to sketch the graph of $f(x)=x^3-3x^2-24x-1$?



No absolute minimum or maximum

Now, write the equation of the line tangent to the curve

$f(x) = x^3 - 3x^2 - 24x - 1$ at $x = 2$.

$$
\begin{aligned}
f(2) &= (2)^3 - 3(2)^2 - 24(2) - 1 \\
&= 8 - 12 - 48 - 1 \\
&= -4 - 48 - 1 \\
&= -52 - 1 \\
&= -53 \quad \therefore f \text{ contains } (2, -53)
\end{aligned}
$$

$f'(x) = 3x^2 - 6x - 24$

$f'(2) = 3(2)^2 - 6(2) - 24$

$= 12 - 12 - 24$

$= -24$ $\therefore$ Slope of tangent line to f at $x = 2$ is -24.

Equation of tangent line to f at $x = 2$ is

$y + 53 = -24(x - 2)$

You Try: Use the techniques from previous examples to sketch the curve.

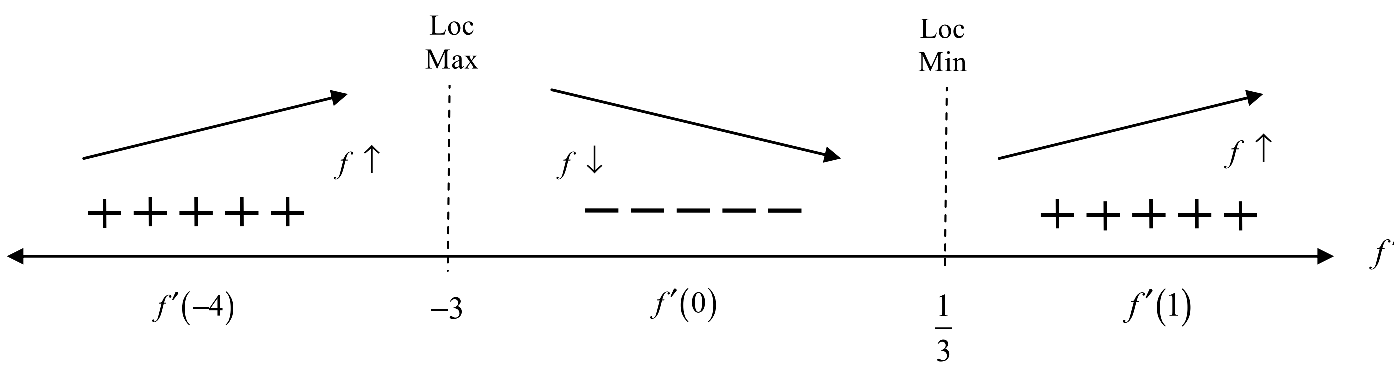
$f(x) = x^3 + 4x^2 - 3x - 6$ Domain $f = (-\infty, +\infty)$

$f'(x) = 3x^2 + 8x - 3$

$f' = 0$ when $3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$

$x = \frac{1}{3}$ or $x = -3$



$f' > 0$ on $(-\infty, -3) \cup \left(\frac{1}{3}, +\infty\right) \therefore f$ is increasing on those intervals

$f' < 0$ on $\left(-3, \frac{1}{3}\right) \therefore f$ is decreasing on $\left(-3, \frac{1}{3}\right)$

f changes from increasing to decreasing at $x = -3 \therefore$ local maximum occurs at $x = -3$

The local maximum value is $f(-3) = (-3)^3 + 4(-3)^2 - 3(-3) - 6$ Local max at $(-3, 12)$

$= 12$

f changes from decreasing to increasing at $x=\frac{1}{3}$ $\therefore$ local minimum occurs at $x=\frac{1}{3}$

The local minimum value is $f\left(\frac{1}{3}\right)=\left(\frac{1}{3}\right)^3+4\left(\frac{1}{3}\right)^2-3\left(\frac{1}{3}\right)-6$

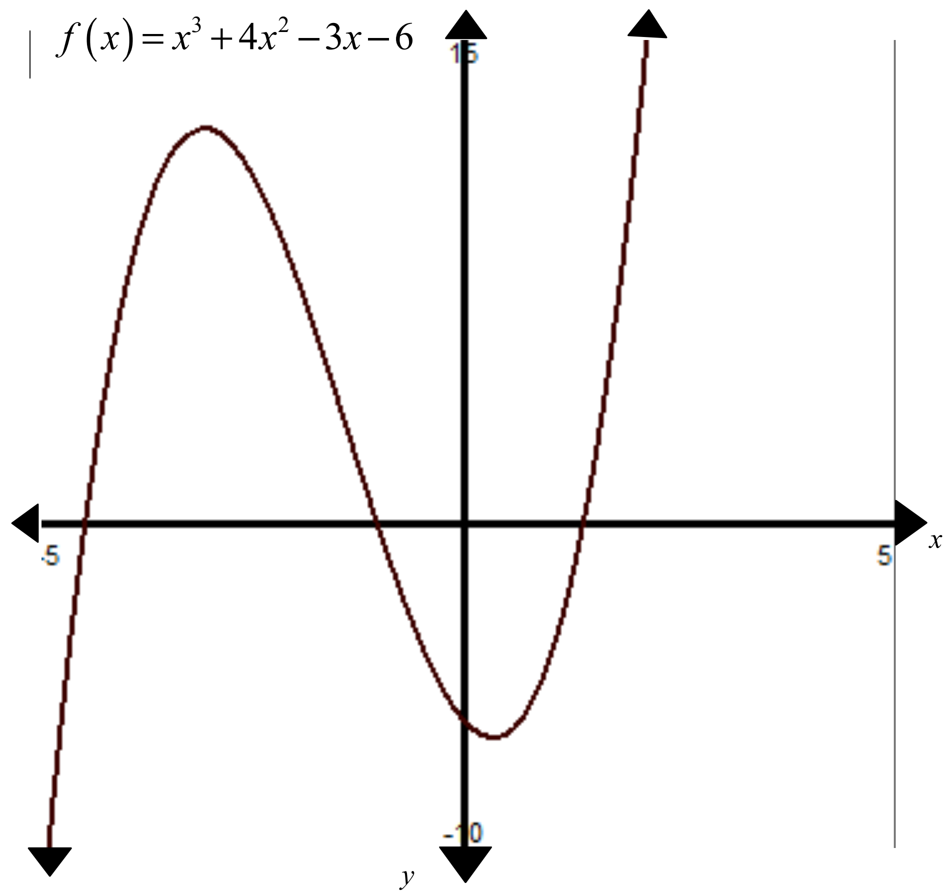
$$=-\frac{176}{27}$$
$$\approx -6.519$$

Local min at $\left(\frac{1}{3},-6.519\right)$

$f''(x)=6x+8$

$f''=0$ when $6x+8=0$

$$6x=-8$$
$$x=-\frac{4}{3}$$



POI

f C.D | f C.U

$-\ -\ -$ | $+\ +\ +$ f''

$f''(-2)$ $-\frac{4}{3}$ $f''(0)$

$f''<0$ on $\left(-\infty,-\frac{4}{3}\right)$ $\therefore f$ is concave down on $\left(-\infty,-\frac{4}{3}\right)$

$f''>0$ on $\left(-\frac{4}{3},+\infty\right)$ $\therefore f$ is concave up on $\left(-\frac{4}{3},+\infty\right)$

f changes concavity at $x=-\frac{4}{3}$ $\therefore$ point of inflection occurs at $x=-\frac{4}{3}$

$$f\left(-\frac{4}{3}\right)=\left(-\frac{4}{3}\right)^3+4\left(-\frac{4}{3}\right)^2-3\left(-\frac{4}{3}\right)-6$$
$$=\frac{74}{27}$$
$$\approx 2.741$$

POI at $\left(-1\frac{1}{3},2.741\right)$

Write the equation of the tangent line to f at $x=1$.

Tangent to $f\left(x\right)=x^3+4x^2-3x-6$ at $x=1$.

$$f\left(1\right)=\left(1\right)^3+4\left(1\right)^2-3\left(1\right)-6$$
$$=1+4-3-6$$
$$=5-3-6$$
$$=2-6$$
$$=-4 \therefore f \text{ contains} \left(1,-4\right)$$

$$f'\left(x\right)=3x^2+8x-3$$
$$f'\left(1\right)=3\left(1\right)^2+8-3$$
$$=3+8-3$$
$$=11-3$$
$$=8$$

$\therefore$ Slope of tangent line to f at $x=1$ is 8.

Equation of tangent line is

$$y+4=8\left(x-1\right)$$